The Mathematics of the WEB

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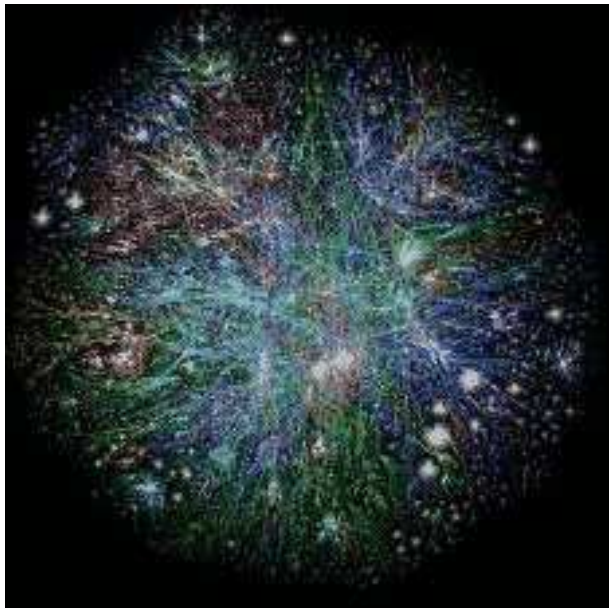
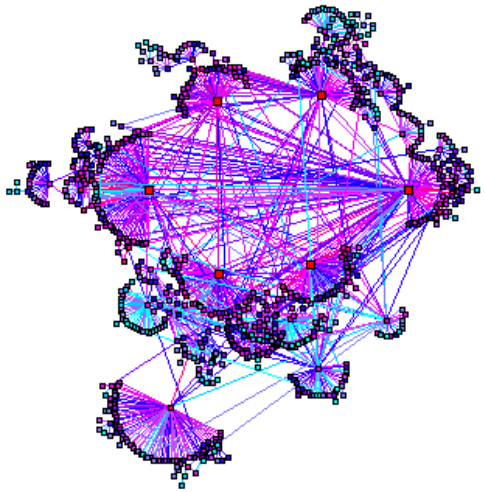
Internet





- The Internet and the Web: full of intersting real problems
- Needs mathematical tools to understand/solve these

Outline

Some Problems

- when you type a query how a search engine (like Google) chooses what to show first?
- Since the web is dynamic, how to efficiently craw the web and to decide which pages to update?
- Which metrics are appropriate to capture important network characteristics and how to calculate them? (how to sample?)
- How to determine the most *influential* node in a network?
- How to discover if there is a path from "s"to "d"?
- How to efficiently search for information and retrieve it (e.g. a movie)?

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- To present a few recent practical problems of the WEB
- To show the mathematical foundations behind the solutions for the problems
- To motivate the students on the subject area
- The lecture is NOT:
  - To present solutions to the problems
  - To develop the mathematical tools

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- Touch on topics such as:
  - Page Ranking
  - Markov Chains
  - Random Walk on Graphs
  - Measurements: Centrality, Clustering, etc
  - Reacommendation Systems
  - P2P scalability
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Ranking Pages

- When you type a query, what happens?
- Search engines return a list of URL's based on a set of key words describing the interest of the client.
- Search engine decides which page to show first, second, etc.
- Problem: how to rank web pages.
- Assign a ranking to pages which is some measure of the quality, or importance of that page . . . $\rightarrow$ the Google search engine

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Ranking Pages
A Graph

- A web page has both forward and backward links. $\rightarrow$ correspondence to a directed graph
- Graph?
  - represents relationships between pairs of objects in a set.
  - Ex: objects are pages, relationship: link between pages
  - Ex: objects are network routers, relationship: data channel connecting routers
  - Many other examples

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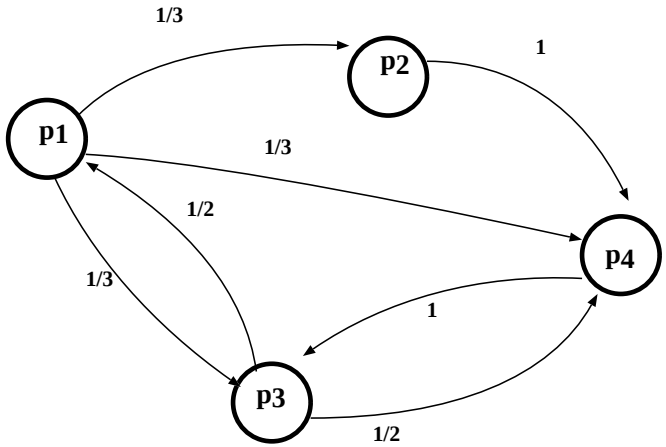
Random Surfer model

- Idea: the rank of a page should derive not just from the number of links to that page but somehow weighted with the ranks of those pages that point to the page.
- Original Google approach: the contribution a page makes to the ranking of the pages it links to should be split evenly among the pages to which it links.



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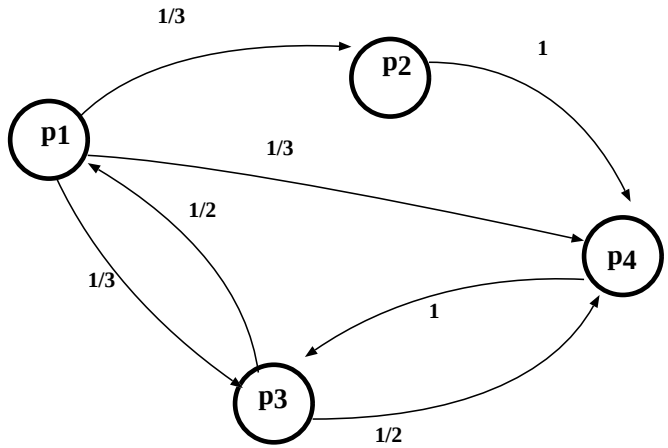

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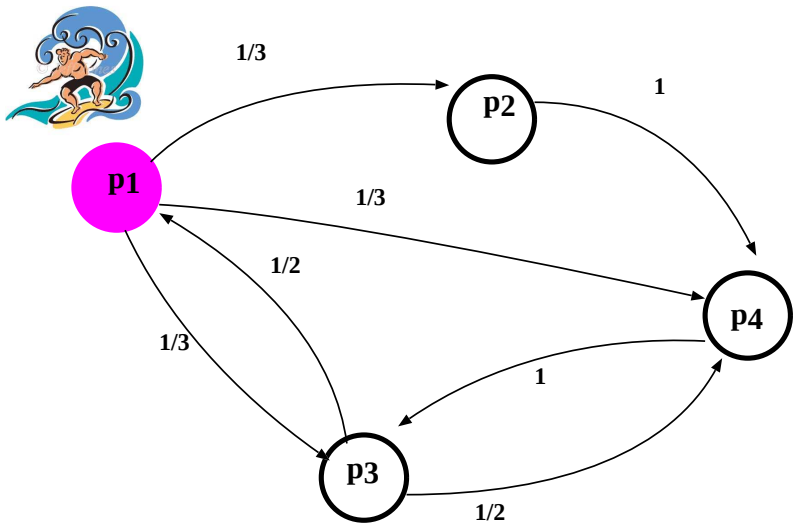
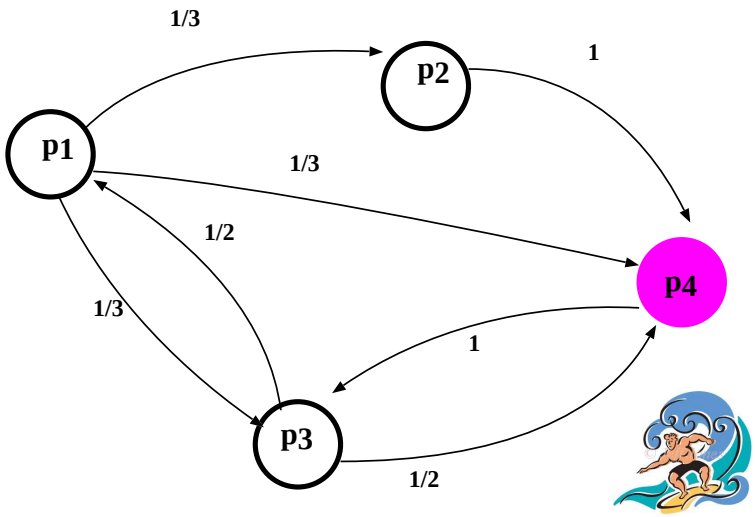


	# of Visits	Fraction of Visits	Probability of Visit
1	1	1	1
2	0	0	0
3	0	0	0
4	0	0	0

Random Walker | Power Method

$$\begin{bmatrix} 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}^{0}$$

Random Surfer model



	# of Visits	Fraction of Visits	Probability of Visit
1	1	0.5	0
2	0	0	0.33
3	0	0	0.33
4	1	0.5	0.33

Random Walker | Power Method

$$\begin{bmatrix} 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}^{1}$$

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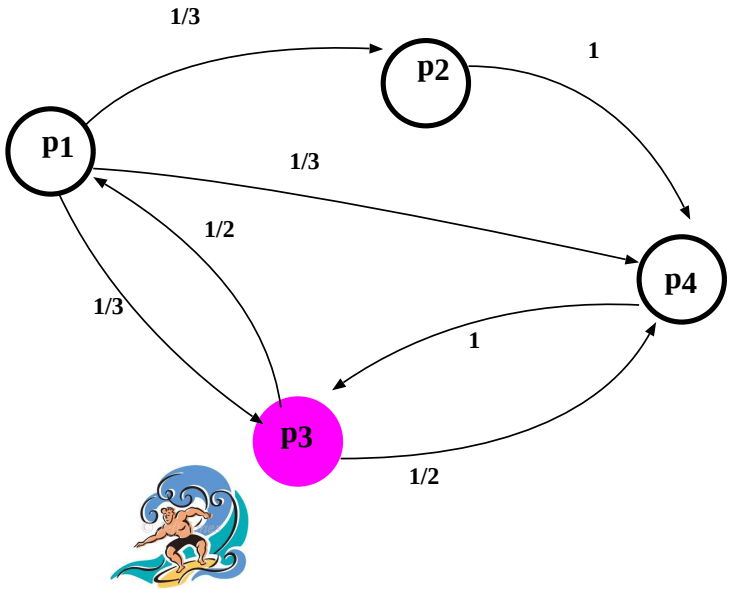
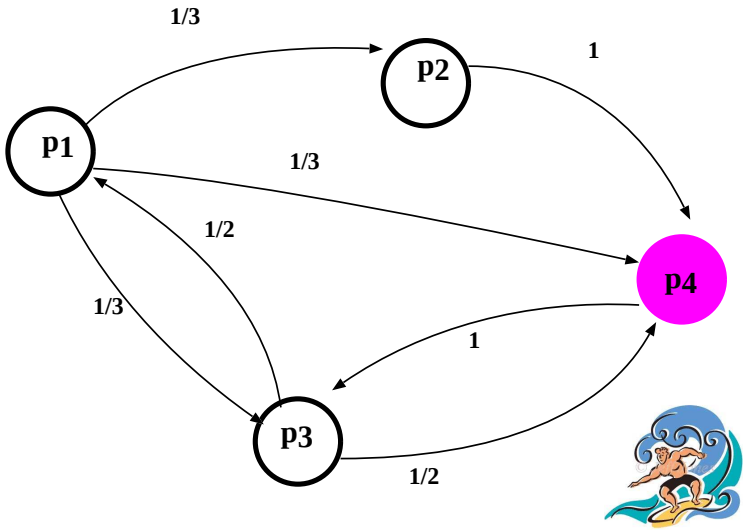


	# of Visits	Fraction of Visits	Probability of Visit
1	1	0.33	0.1667
2	0	0	0
3	1	0.33	0.3333
4	1	0.33	0.5

Random Walker — Power Method

$$\begin{bmatrix} 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}^2$$

Random Surfer model



	# of Visits	Fraction of Visits	Probability of Visit
1	1	0.25	0.1667
2	0	0	0.0556
3	1	0.25	0.5556
4	2	0.5	0.2222

Random Walker — Power Method

$$\begin{bmatrix} 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}^3$$

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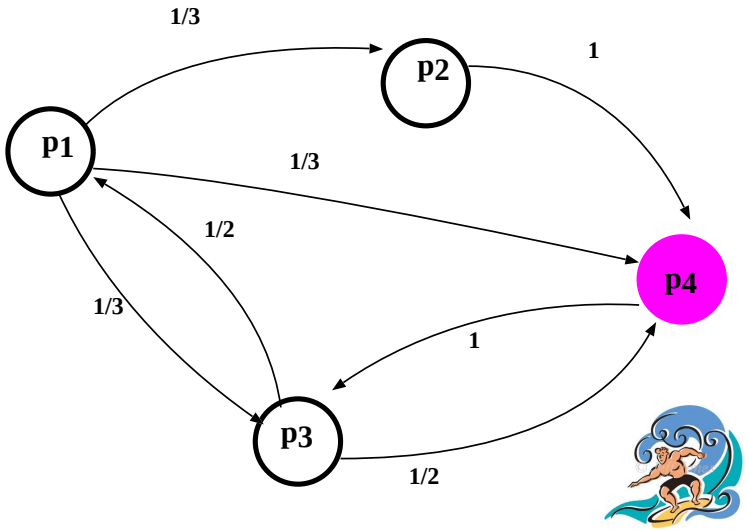

	# of Visits	Fraction of Visits	Probability of Visit
1	200	0.20	0.20
2	67	0.06	0.06
3	400	0.40	0.40
4	333	0.33	0.33

Random Walker — Power Method

$$\begin{bmatrix} 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}^{1000}$$

Random Surfer model

- Let:
  - R_i be the ranking of a page i
  - n_i be the number of forward links out of page i
  - B_i be the set of pages that link to page i.
- The above ideas can be captured in the equation:

$$R_i = \sum_{j \epsilon B_i} R_j \frac{1}{n_j}$$

- This set of equations can be viewed as a “random web surfer” who visits pages and is equally likely to pick any of the forward links from the current page.
- These equations are the same as for a Markov chain!!!

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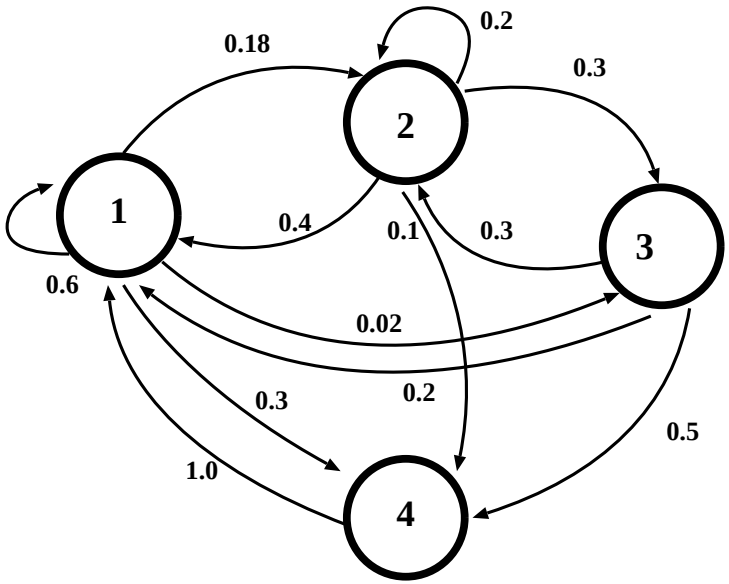
Everything you wanted to know about MCs and was afraid to ask

- state transition diagram
- directed graph.
- assign *transition probabilities* to arcs or *transition rates*.
- what a state represents? · · · Notion of state variables



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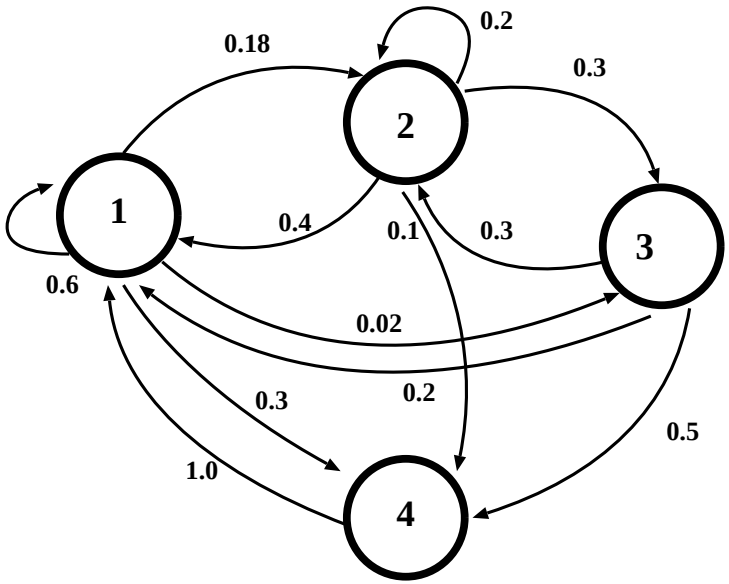

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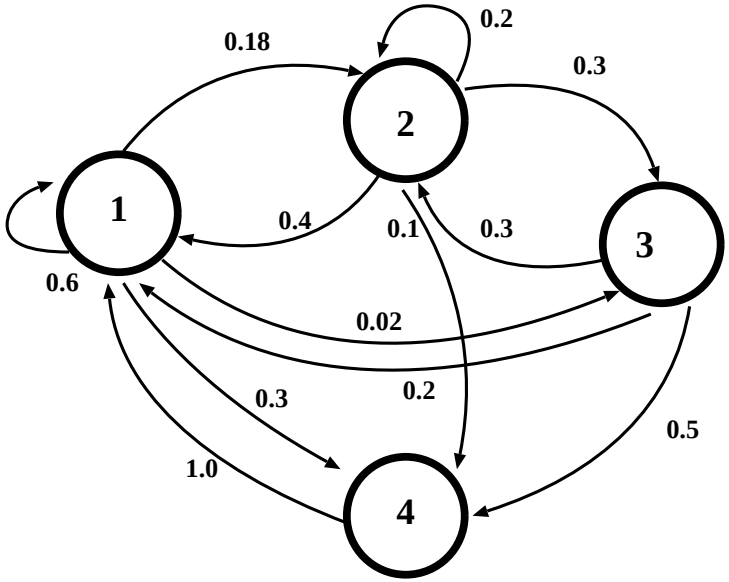

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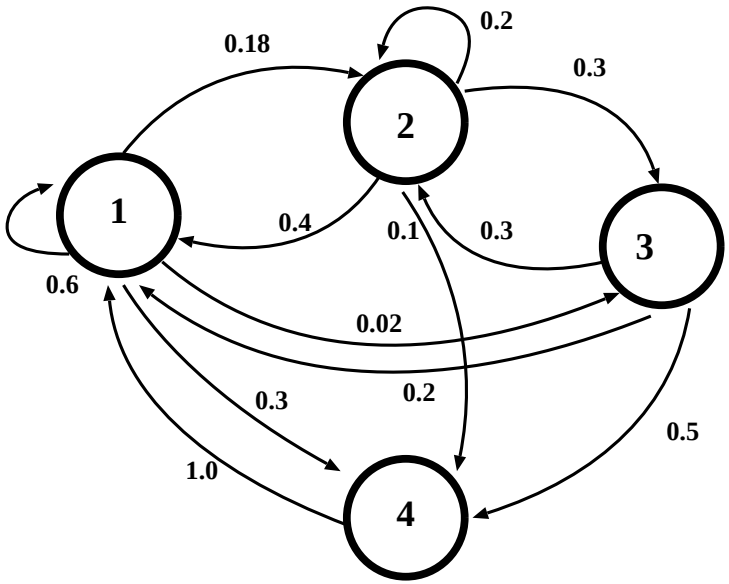

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Measures of interest

- What can we calculate?
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- fraction of visits to a state
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Simple Example

- President of a large corporation travels every friday from one branch of the company to the other, and stays in that branch for the whole week.
- each branch is located in a different city. (eg. SP, Rio, MG, Brasilia, São Luis, Belem, Salvador, PA, Manaus)
- President only choses one particular company and choses the next city to visit if there is a non-stop flight from the current city to the next.
- If there are more than one choice for the next city, she chooses the next city at random.
- you know that the President has been doing this for a long time but you do not know whwere the President curently is.
- you want to meet the President, but you must be at the city she is in to succeed. Question: which city do you choose?

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- The page ranks are proportional to the state probabilities.
- Solution: Power method (iterative method)

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$$r_i = d \sum_{j \in \mathcal{I}_i} r_j \frac{1}{c_j} + (1-d)T[i] \quad \forall i$$

- Heinrich Hertz

  One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.

  (regarding Maxwell's equation)

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path exists?

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