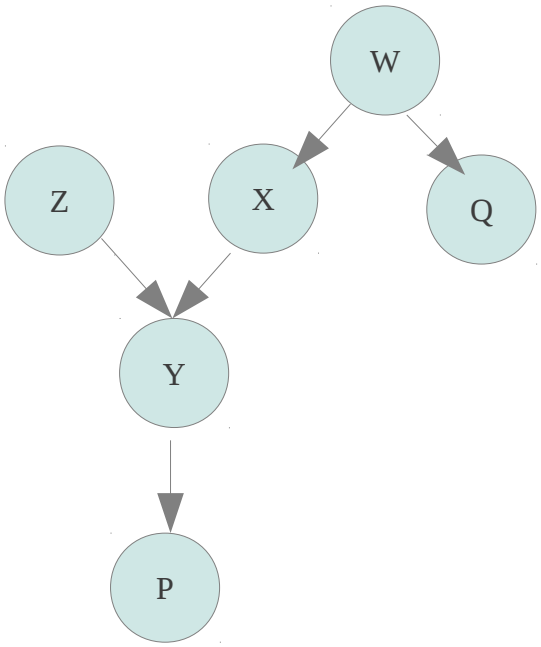
Homework 4: Graphical Models

Question 1: Bayesian Networks (10 points)

Given the Bayesian network below, answer TRUE or FALSE and justify ($\perp\!\!\!\perp$ means “is independent”)

1) $Z \perp\!\!\!\perp Z$
2) $Z \perp\!\!\!\perp Z \mid Z$
3) $Y \perp\!\!\!\perp W$
4) $Y \perp\!\!\!\perp W \mid X$
5) $X \perp\!\!\!\perp Q \mid W$
6) $Q \perp\!\!\!\perp X, Y, Z, P \mid W$
7) $Z \perp\!\!\!\perp X, W, Q \mid \varnothing$
8) $Z \perp\!\!\!\perp X, W, Q \mid P$
9) $Z, Y, P \perp\!\!\!\perp W, Q \mid X$
10) $Z, Y, P \perp\!\!\!\perp W, Q \mid \varnothing$



Question 2: Markov Random Fields (90 points)

In this question we study and implement an algorithm for image de-noising.

Read section 8.3.3 of the textbook and answer the following questions

1) (10 points) write down an expression for the difference in the values of the energy associated with the two states of a particular variable x_j, with all other variables held fixed, and show that it depends only on quantities that are local to x_j in the graph.

2) (10 points) what is the output of the algorithm if h=0 and β=0? Why? Use 1) to prove your answer.

3) (70 points) implement the image de-noising algorithm (should not take more than 150 lines of code, in Matlab). Given the image in the next page (smilenoise.txt and smilenoise-space.txt), set β=1 and η=0.9, and run your algorithm, raster scanning through the image at least 5 times. Print the output.

The image without noise is shown in the last page (smile.txt and smile-space.txt). What is the fraction of inverted bits in the image in the next page? And in the final image, after de-noising?