# CORRECTION TO "A FAMILY OF FOLIATIONS WITH ONE SINGULARITY"

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Abstract. We fix a mistake in the argument leading to the proof that the family of foliations introduced in the paper does not have an algebraic solution apart from the line at infinity.

We use throughout the notation introduced in section 5 of our paper. A mistake was introduced in the paper by our assumption in Proposition 5.1 that $\phi_m = 1$, because it requires that we divide all the coefficients of the algebraic solution $f$ by $\phi_m$. However, in doing that we get a polynomial whose coefficients do not belong to $R$, which precludes the divisibility arguments used to get contradictions in Propositions 5.1 and 5.3 and in Theorem 5.5. We show here that it is possible to arrive at the same contradictions without requiring $f$ to be in $R[y, z]$.

We will assume that

$$A = \alpha y^{k-3}, \quad B = \beta y^{k-1} \quad \text{and} \quad q = \rho y^2,$$

since this is the only case needed for Theorem 5.5 and it simplifies the equations. We will also assume that $k \geq 7$, to avoid a multiplicity of cases. Examples of smaller degree can be handled using a computer algebra system.

**Proof of Proposition 5.1.** The only point in the proof where a divisibility argument is used is at the very end of the proof, after the equation

$$z^{k-2}\phi'_{m+2} - 2A\phi_{m+2} = -(d-m)h. \tag{1}$$

Note that there was a typo on the first term of (1). Since we have already shown that $e_{m+2} = m - 2$, it follows that

$$\deg(\phi_{m+2}) = (m+2) - (m-2) = 4.$$

Set $\phi_{m+2} = e_4 z^4 + e_3 y z^3 + e_2 y^2 z^2 + e_1 y^3 z + e_0 y^4$. The terms $-2\alpha e_i y^{k+1-i} z^i$ on the left hand side of (1) have no correspondent on the right side for $i = 1, \ldots, 4$ and the same holds for $4e_0 y^3 z^{k-2}$. Thus, $\phi_{m+2} = 0$, which gives the desired contradiction because $d \neq m$ and $h \neq 0$ by hypothesis. □

**Proof of Proposition 5.3.** Divisibility arguments are used twice in the proof of this proposition. The first time is right at the beginning, where we assume that $\mu(m) = \varepsilon_0(m) = \varepsilon_2(m) = m$. However, in this case, the equation that results from (5.7) is (1) and we have already shown that it leads to a contradiction. The second

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place where a divisibility condition intervenes is at the end of the proof. But having proved that $e_j = 3m - 2j$, we know from Corollary 3.2 that $e_d = 3m - 2d = 0$. In particular, $e_{d-1} = e_d + 2 = 2 > 0$; so that $\varepsilon_0(d-1) = 2$, $\varepsilon_1(d-1) = 1$ and $\varepsilon_1(d-1) + \sigma = 0$. Thus, $\underline{\mu(d-1)} = 0$. Since $\theta_{d+1} = \varpi_{d+1} = 0$, it follows from (5.7), with $r = d-1$, that $\overline{\theta_3(d-1)} = -\alpha\psi_{m+1}\psi_d = 0$, which is a contradiction by Lemma 5.1 and Corollary 3.2. □

**Proof of Theorem 5.5** It follows from Proposition 5.4 that $e_j = 2m - j$, so that $e_{m+1} = m - 1$ and $e_{m+2} = m - 2$, while $\deg(\phi_{m+1}) = 2$ and $\deg(\phi_{m+2}) = 4$. Thus, equation (5.6) with $r = m$ gives

(2)

$$(m-d)h + y^{k-3}(\beta y^2\phi_{m+1} + 2\alpha\phi_{m+2} - \alpha\phi_{m+1}^2) = z^{k-2}(\rho y^2\phi'_{m+1} + \phi'_{m+2} - \phi'_{m+1}\phi_{m+1}).$$

Equating the coefficients of $y^{k-2}z^3$ and $z^{k+1}$ on both sides of (2), we get

$$2e_1\alpha - 2s_0s_1\alpha = 0 \quad \text{and} \quad c(m-d) = e_1 - s_0s_1.$$

Since $\alpha \neq 0$ the first equation gives $e_1 = s_0s_1$. Substituting this into the second equation above we get $c(m-d) = 0$, which gives a straightforward contradiction.

□

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